

Modeling with Logarithms

I. Solve for x:

1.  $e^x = 3$       2.  $\log(3x) = 2$       3.  $2 \cdot \ln(x) - 3 = 15$       4.  $5 \cdot 10^{2x} = 5000$       5.  $6 + 4^x = 70$

II. Physics: Sound Intensity: Logarithms to the base 10 are used in the decibel (dB) scale for measuring loudness. If the intensity of the sound in watts per square meter ( $\frac{w}{m^2}$ ) is  $I$ , the decibel level dB is given by  $\text{dB} = 10 \log(I \times 10^{12})$  decibels.

6. The intensity of a whisper is measured at  $300 \times 10^{-12} \text{ w/m}^2$ . Find the decibel level of the whisper.

7. The sound level of a rock band is measured at 115 dB. Find the intensity of the sound in  $\text{w/m}^2$ .

III. Physics: Earthquake Magnitude: On the Richter scale, the magnitude  $M$  of an earthquake depends on the amount of energy,  $E$  (measured in ergs), released by the earthquake as follows:  $M = \frac{2}{3} \log \frac{E}{10^{11.8}}$

8. In 1985, an earthquake hit Mexico City and measured 8.1 on the Richter scale. Find the amount of energy,  $E$ , released by this earthquake.

9. In 1976, an earthquake that released about  $8 \times 10^{19}$  ergs of energy occurred in El Salvador. Find the magnitude,  $M$ , of this earthquake to the nearest tenth.

IV. Physics: Newton's Law of Cooling: This law states that the temperature difference between an object ( $T$ ) and its surroundings ( $T_s$ ) decreases exponentially as a function of time ( $t$ ) according to the following:  $T(t) = T_s + (T_0 - T_s)e^{-kt}$ , where  $T_0$  is the initial temperature of the object, and  $-k$  is our constant of variation representing the constant rate of decrease in the temperature difference.

10. When a jug of milk is taken out of a refrigerator, its temperature is  $40^\circ\text{F}$  and set in a  $70^\circ\text{F}$  room. One hour later, its temperature is  $50^\circ\text{F}$ . What is the temperature of the milk after 2 hours?

11. When will your jug of milk in #9 reach a temperature of  $65^\circ\text{F}$  ?

12. A cup of cocoa has cooled from  $92^{\circ}\text{C}$  to  $50^{\circ}\text{C}$  after 12 minutes in a room at  $22^{\circ}\text{C}$ . How long will the cup take to cool to  $30^{\circ}$  ?

V.Chemistry: pH Factor: The acidity of a water-based solution is measured by the concentration of hydrogen ions ( $H^+$ ) in the solution (in moles per liter). The measure of acidity is the pH factor with the relationship  $\text{pH} = -\log[H^+]$  More acidic solutions have higher hydrogen-ion concentrations and lower pH values.

13. Baking soda has a pH value of 8.4. What is its hydrogen-ion concentration?

14. What is the pH level of human blood if its hydrogen-ion concentration is  $3.98 \times 10^{-8}$  moles per liter?

VI.Finance: Compound Interest Compound Interest:  $A = P \left(1 + \frac{r}{n}\right)^{nt}$ , Continuously:  $A = P \cdot e^{rt}$

15. What interest rate, compounded annually, do you need in order to grow a \$1000 investment to \$5000 in 20 years?

16. How long will it take you to triple an investment of \$300 if it is invested at 5.5% compounded continuously?

17. How long will it take a \$1500 investment to grow to \$2000 if it is invested at 3.72% compounded monthly?

18. What interest rate, compounded quarterly, is needed to grow an \$800 investment to \$2000 in 10 years?

19. What interest rate, compounded continuously, is needed to grow an \$800 investment to \$2000 in 10 years?

HW 8-5  
Secondary 3

Name: \_\_\_\_\_  
Date: \_\_\_\_\_ Period: \_\_\_\_\_

Answers: 1.  $x = \ln 3$

2.  $x = 100/3$

3.  $x = e^9$

4.  $x = 3/2$

5.  $x = 3$

6.  $\approx 25$  dB

7.  $10^{-0.05}$  w/m<sup>2</sup>

8.  $8.9 \times 10^{23}$  erg

9. magnitude 5.4

10.  $\approx 56.7^\circ\text{F}$

11.  $\approx 4.42$  Hours

12.  $\approx 28.41$  minutes

13.  $\approx 3.98 \times 10^{-9}$  moles/liter

14. pH  $\approx 7.4$

15.  $r \approx 8.38\%$

16.  $t \approx 19.97$  years

17.  $t \approx 7.75$  years

18.  $r \approx 9.27\%$

19.  $r \approx 9.16\%$