January 12, 2015

13 $a = 8$

$r = 1/2$

$n = ?$

8

4

2

1

$1 = 8\left(\frac{1}{2}\right)^n$

$\frac{1}{8} = \left(\frac{1}{2}\right)^n$

$\left(\frac{1}{2}\right)^3 = \frac{1}{2}^n$

$n = 3$

$$S = \frac{8\left(1 - \frac{1}{2}^{3+1}\right)}{1 - 1/2}$$

15 matches

5 $-12 - 4 - \frac{4}{3} - \dots - \frac{4}{243}$

$a = -12$

$r = \frac{1}{3}$

$n = 6$

$$\frac{-4}{243} = -12\left(\frac{1}{3}\right)^n$$

$$\frac{-\cancel{4}\,1}{3 - \cancel{12}(243)} = \left(\frac{1}{3}\right)^n$$

$$\frac{1}{729} = \frac{1}{3}^n$$

$$\left(\frac{1}{3}\right)^6 = \frac{1}{3}^n$$

$n = 6$

$$S = \frac{-12\left(1 - \left(\frac{1}{3}\right)^{6}\right)}{1 - 1/3}$$

10

$a = 1$

$r = 1/3$

$n = 4$

$$S = \frac{1 \cdot \left(1 - \left(\frac{1}{3}\right)^{4+1}\right)}{1 - 1/3}$$

$$S = \frac{121}{81} \approx 1.49$$

11

$a = 1000$

$r = .8$

$n = 6$

$$S = \frac{1000(1 - .8^{6+1})}{1 - .8}$$

16

$a = 5000$

$r = 1.04$

$n = 9$

$$S = \frac{5000(1 - 1.04^{9+1})}{1 - 1.04}$$

7-3 Exponential Review

I can apply exponential properties and use them

I can model real-world situations using exponential functions

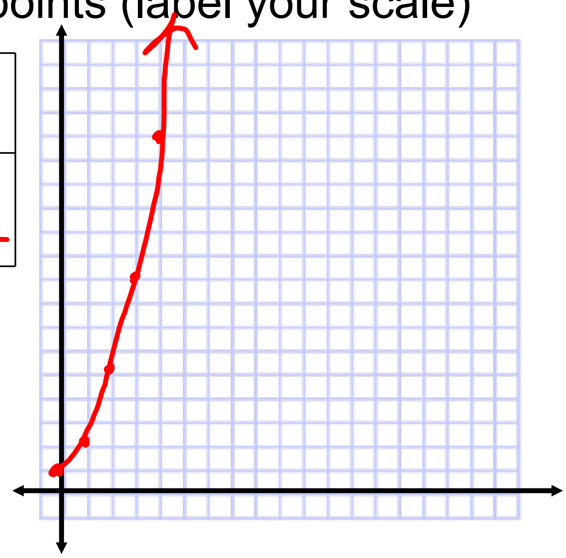
Warm-Up

1. Find the next three terms in the sequence

   2, 6, 18, 54, 162, 486, 1458

2. Fill in the table, then plot the points (label your scale)

n	0	1	2	3	4	5
f(n)	1	2	4	8	16	32



If we connected the points, what do you notice about the graph?

Have you ever seen a graph like this before?

exponential

EXPONENTIAL FUNCTION

$$f(x) = a(b)^x$$

Exponent

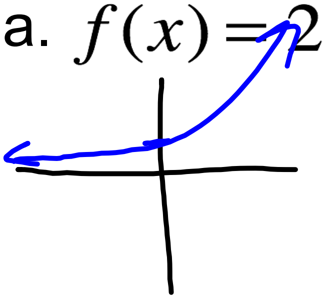
Initial Value
(y-intercept)

Base
(Multiplier)

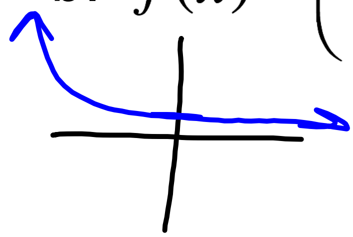
$\cancel{x^2}$

Graph the following functions on a calculator and sketch.
Be sure to plot the y-intercept

a. $f(x) = 2^x$



b. $f(x) = \left(\frac{1}{2}\right)^x$



What did you notice about the graphs and their equations?

growth

decay

$$f(x) = a(b)^x$$

Exponential Growth and Decay

When $b>1$, the function represents **exponential growth**

When $0<b<1$, the function represents **exponential decay**

Determine whether each function represents growth or decay

a. $f(x)=13\left(\frac{1}{3}\right)^x$

decay

b. $g(x)=\left(\frac{\cancel{3}}{2}\right)^x$ 1.5

growth

Write one equation that represents growth and one that represent decay

$$f(t) = a(1 \pm r)^t$$

John researches a baseball card and find that it is currently worth \$3.25. However, it is supposed to increase in value 11% per year. pg 686 $f(t) = a(1 \pm r)^t$

a) Write an exponential equation to represent this situation

$f(t) = 3.25(1+.11)^t$

b) How much will the card be worth in 10 years?

$f(10) = 3.25(1.11)^{10} = \$9.23$

c) Use your graphing calculator to determine in how many years will the card be worth \$26.

$26 = 3.25(1.11)^t$

about 20 years

You Try!

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at $2765 depreciates at a rate of 30% per year. pg 704 $f(t)=a(1\pm r)^t$

a) Write an exponential equation to model this situation

$f(t)=2765(1-.3)^t$

b) How much will this computer be worth in 5 years?

$f(5)=2765(.7)^5=\$464.71$

c) Use your graphing calculator to determine in how many years will the computer be worth $350.

$350=2765(.7)^t$

5.79 years

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

$P(t) = 4000(1+.026)^t$

$t=25 = 7599$ ppl

$t=50 = 14{,}435$ ppl

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

$200{,}000 = 4{,}000(1.026)^t$

$t=152$ 2102

$f(t) = a\left(\frac{1}{2}\right)^{t/n}$

The half-life of Carbon-14 is 5700 years. If a fossil decayed from 15 grams to 1.875 grams, how old is the fossil? (use your calculator)

$f(t) = 15\left(\frac{1}{2}\right)^{t/5700}$

$1.875 = 15\left(\frac{1}{2}\right)^{t/5700}$

$3(5700) = 17,100$

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

P is the principal

r is the annual interest rate

n is the number of compounding periods per year

t is the time in years

Write an equation then find the final amount for each investment.

a. \$1000 at 8% compounded semiannually for 15 years

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

You Try!

b. \$1750 at 3.65% compounded daily for 10 years

Using a calculator, determine how many years it will take for the amount to reach \$4000.

Investigate the growth of $1 investment that earns 100% annual interest (r=1) over 1 year as the number of compounding periods, n, increases.

Compounding schedule	n	$1\left(1+\frac{1}{n}\right)^n$	Value of A
annually	1		
semiannually	2		
quarterly	4		
monthly	12		
daily	365		
hourly	8760		
every minute	525600		

What does the value of A approach?

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x) = e^x$ for

a. $x = 2$

b. $x = \frac{1}{2}$

c. $x = -1$

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

Continuous Compounding Formula

If P dollars are invested at an interest rate r, that is compounded continuously, then the amount, A, of the investment at time t is given by

$$A(t) = Pe^{rt}$$

A person invests $1550 in an account that earns 4% annual interest compounded continuously.

a. Write an equation to represent this situation

b. Using a calculator, find when the value of the investment reaches $2000.

Pg 730 $A(t) = Pe^{rt}$

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*

CHECK:

-What is the difference between growth and decay?

-What does each piece of the equation $y = a(b)^x$ represent?