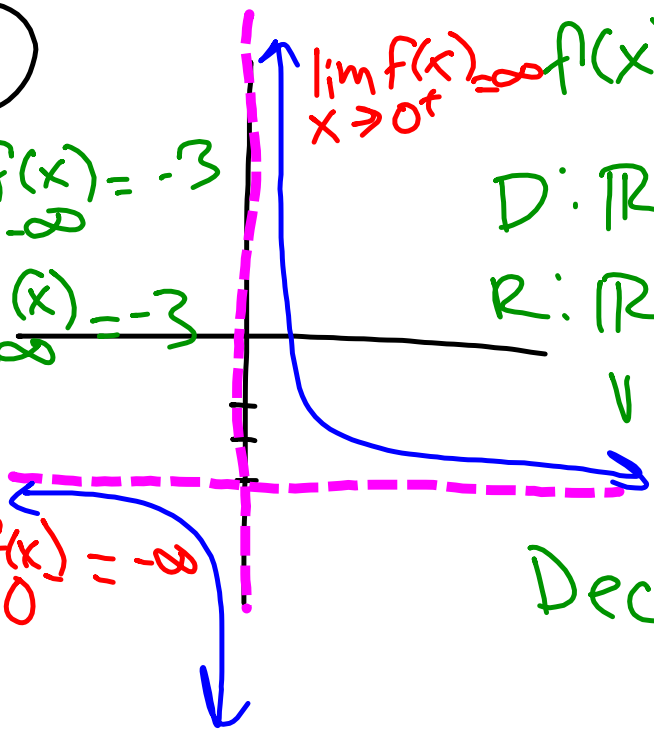
15

$f(x) = \frac{1}{x} - 3$

$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -3$

$\lim_{x \to \infty} f(x) = -3$



$\lim_{x \to 0} f(x) = -\infty$

D: $\mathbb{R}$, $x \neq 0$

R: $\mathbb{R}$, $y \neq -3$

VA $x = 0$

HA $y = -3$

Dec: $(-\infty, 0) \cup (0, \infty)$

# 6-2: Asymptotes of Rational Functions
# Book 8.2

Objectives:

1. I can find the x and y intercepts of a rational function
2. I can find the vertical and horizontal asymptotes of a rational function
3. I can find the holes of a rational function
4. I can analyze a graph of a rational function
5. I can graph a rational function by hand

## Review of Vertical Asymptotes

$$f(x) = \frac{2}{x+3}$$

$x + 3 = 0$

VA $x = -3$

Set the denominator = 0, then solve for x

## Find the vertical asymptotes:

a. $y = \dfrac{3x-5}{x^2-4}$ $(x+2)(x-2)$

$x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

b. $y = \dfrac{2x^3}{x-5}$

$x = 5$

c. $y = \dfrac{5x}{x+2}$ $x = -2$

# X and Y Intercepts

Y intercepts, x = 0

$$f(x)=\frac{3x-12}{x^2-5x-6}$$

$$\frac{-12}{-6}$$

$(0,2)$

X intercepts, y = 0

$$f(x)=\frac{3x-12}{x^2-5x-6}$$

$$0=\frac{3x-12}{x^2-5x-6}$$

$3x-12=0$

$3x=12$

$x=4$

$(4,0)$

Find the x and y intercepts of the following functions:

$$f(x) = \frac{x^2 - 2x - 3}{x + 2}$$

x-int

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3, -1$

$(3,0)\ (-1,0)$

y-int $x=0$

$\frac{0-0-3}{0+2} = -3/2$

$(0, -3/2)$

$$f(x) = \frac{3x - 5}{x^2 - 5x + 6}$$

y-int $x=0$

$(0, -5/6)$

x-int $y=0$

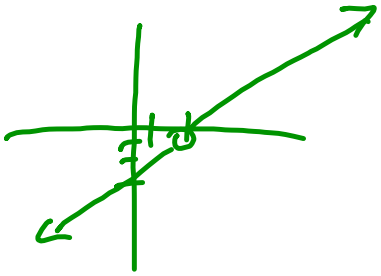
$0 = 3x - 5$

$(5/3, 0)$

# Asymptotes:

**check for holes before VA!! (by reducing the fraction if possible)**

$$f(x) = \frac{(x-3)\cancel{(x-2)}}{\cancel{(x-2)}}$$



vertical (VA): caused by dividing by 0

the graph approaches $-\infty$ *or* $\infty$

on each side of the asymptote

to find the asymptote set den = 0 and solve

Identify any holes, then find all vertical asymptotes

$$f(x)=\frac{x^2-9}{x^2-5x+6}$$

$$\frac{(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}$$

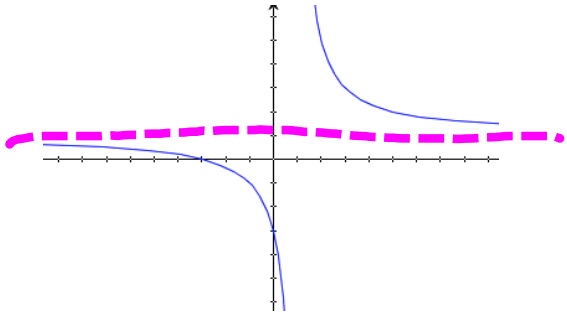
hole @ $x=3$

VA: $x=2$

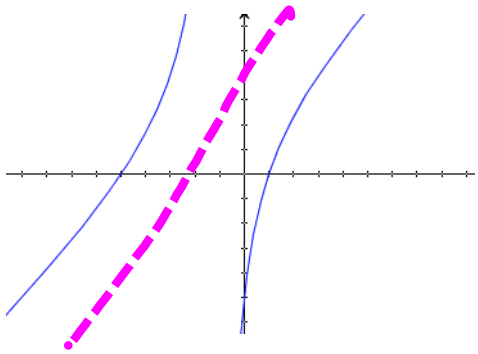
# Horizontal Asymptotes

Look at the graphs, see if you can find the horizontal asymptote. Are there any patterns?

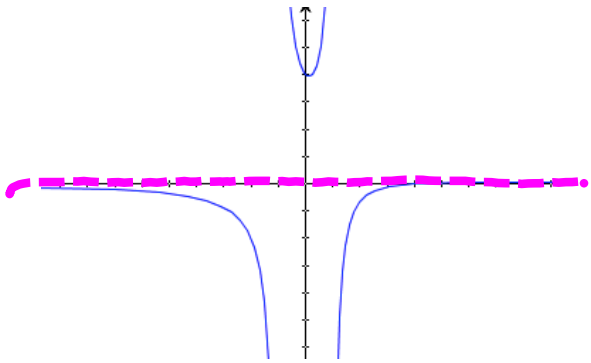
$$f(x)=\frac{x+3}{x-1}$$



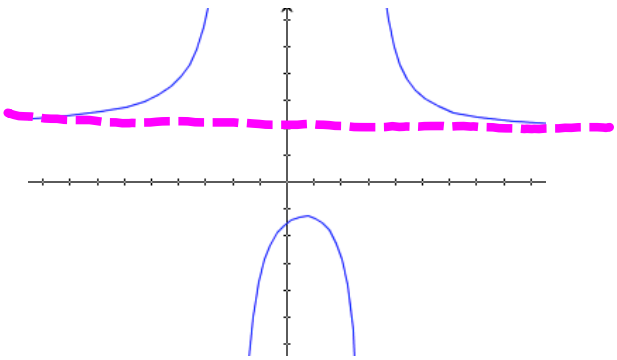
$$f(x)=\frac{(x+5)(x-1)}{x+1}$$



$$f(x)=\frac{x-4}{(x+1)(x-1)}$$



$$f(x)=\frac{2x^2-3x+9}{x^2-x-6}$$



end behavior: (horizontal (HA) or oblique (OA))

to find the asymptote - compare the degrees of the numerator and denominator if:

memorize *

top heavy (OA): oblique
bottom heavy (HA): $y = 0$
equal (HA): divide coefficients

oblique: (more later)

Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

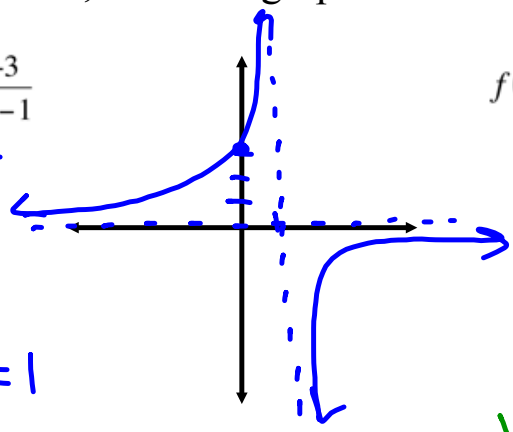
$f(x) = \frac{-3}{x-1}$

x-int –

y-int (0,3)

VA: x=1

HA: y=0



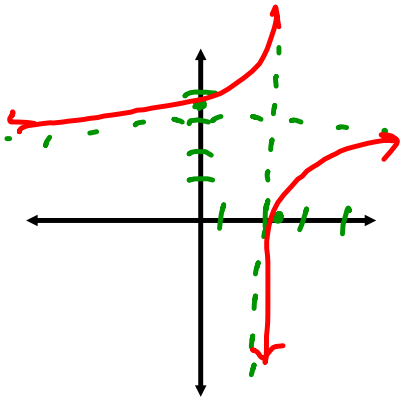
$f(x) = \frac{3x-7}{x-2}$

x-int (7/3,0)

y-int (0, 7/2)

VA x=2

HA y=3



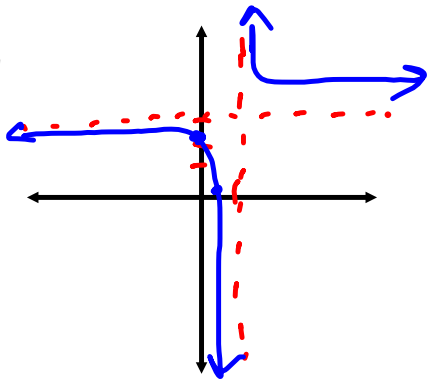
$f(x) = \frac{3x-2}{x-1}$

x-int (2/3,0)

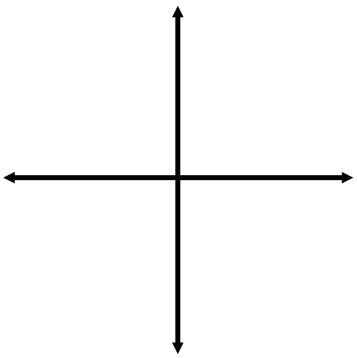
y-int (0,2)

VA: x=1

HA: y=3



$f(x) = \frac{x^2+2}{x^2+1}$



## End Behavior (Oblique/Slant) Asymptotes

top heavy rational functions have oblique asymptotes (end behavior models)

to find the degree of the end behavior **model (EBM)** - divide the leading terms and reduce

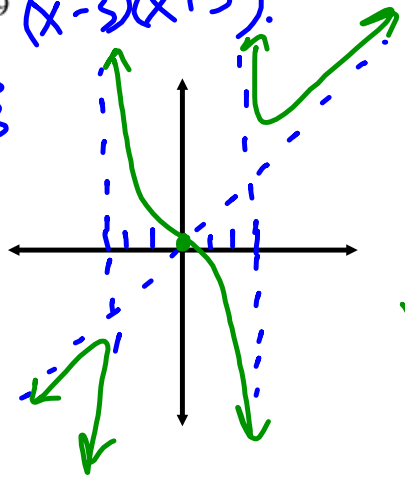
the ends of $\dfrac{3x^5 - 4x^2 + 5}{2x^3 - 5x + 4}$ will behave like $\dfrac{3x^5}{2x^3} = \boxed{\dfrac{3x^2}{2}}$

**Ex. 5** Find asymptotes/EBM, holes and graph.

a. $f(x)=\frac{x^3}{x^2-9}$ $\frac{x^3}{(x-3)(x+3)}$

VA: $x=3$
$x=-3$

EBM:
$y=x$



b. $f(x)=\frac{x^2-9}{x^2-5x+6}$ $\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-2)}$

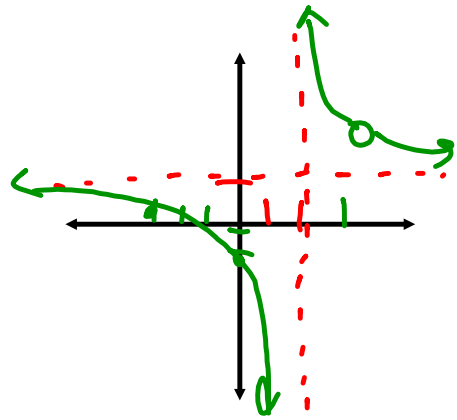
hole @ $x=3$

VA $x=2$

HA $y=1$

y-int $(0,-3/2)$

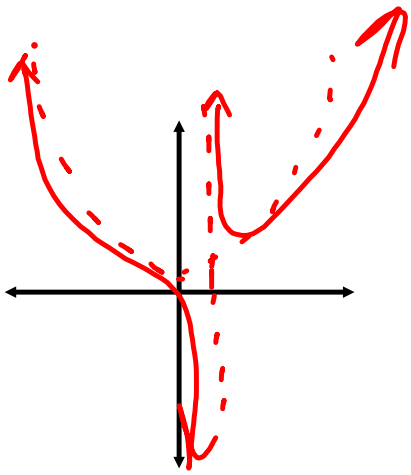
x-int $(-3,0)$



c. $f(x)=\frac{x^3-3x^2+3x+1}{5(x-1)}$

VA: $x=1$

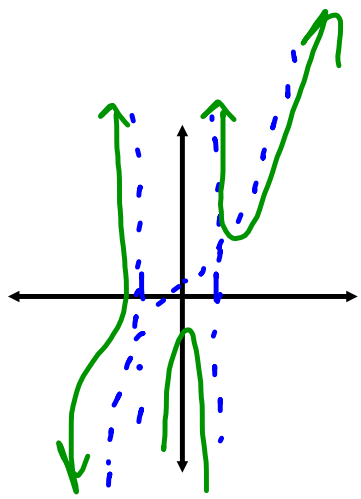
EBM:
$y=\frac{1}{5}x^2$



d. $f(x)=\frac{2x^5+x^2-x+1}{x^2-1}$

V.A. $x=1,-1$

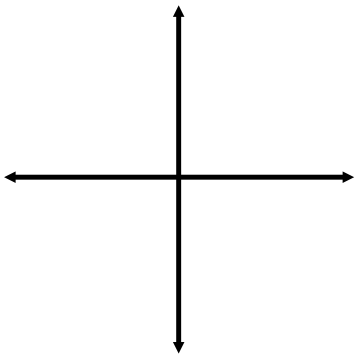
EBM.
$y=2x^3$



Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x) = \frac{x-1}{x^2 - x - 12}$$

Domain
Range
x-intercepts
y-intercepts
VA
HA
Increasing
Decreasing
Continuous
Asymptote Behavior



End Behavior