$3x^2 + x - 24$

$(3x - 8)(x + 3)$

$3xy^2(2x^2 + 5x - 12)$ $\begin{matrix} -24 \\ +5 \end{matrix}$

$(2x^2 + 8x - 3x - 12)$

$2x(x+4) - 3(x+4)$

$3xy^2(x+4)(2x-3)$

3.1 Zeroes of a Polynomial

Book Pages: 371-372

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem

Divide the following polynomials

$$x+4\overline{)3x^2+7x-20}$$

-4	3	7	-20
	↓	-12	20
	3	-5	0

$3x-5$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

3	2	-5	7	-3	1
	↓	6	3	30	81
	2	1	10	27	82

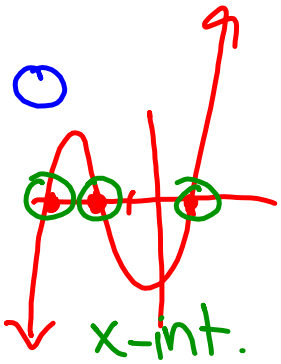
$2x^3+x^2+10x+27+\frac{82}{x-3}$

Identify the zeros of the following and explain what that means graphically.

$$f(x) = (x+2)(x-1)(x+3) = 0$$

$x+2=0$ $\quad$ $x-1=0$ $\quad$ $x+3=0$

$x=-2$ $\quad$ $x=1$ $\quad$ $x=-3$



Write the function in standard form and state the relationship between the degree and zeros of the function

$(x^2+x-2)(x+3)$

$x^3+x^2-2x+3x^2+3x-6$

x^3+4x^2+x-6

deg $=3$

zeros $=3$

Remainder Theorem: $(x-2)$

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

$p(2) =$

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

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Example 3 **Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.**

(A) $p(x) = x^3 + 3x^2 - 4x - 12$; $(x + 3)$

Use synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

Since the remainder is 0, $x + 3$ is a factor.

Write $q(x)$ and then factor it.

$q(x) = x^2 - 4 = (x + 2)(x - 2)$

So, $p(x) = x^3 + 3x^2 - 4x - 12 = (x + 2)(x - 2)(x + 3)$.

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(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5$; $(x + 1)$

Use synthetic division.

-1	1	-4	-6	4	5
	↓	-1	5	1	-5
	1	-5	-1	5	0

Since the remainder is 0, $(x + 1)$ is a factor. Write $q(x)$.

$q(x) = x^3 - 5x^2 - x + 5$

Now factor $q(x)$ by grouping.

$q(x) = (x^3 - 5x^2) + (x + 5)$

$= x^2(x - 5) - 1(x - 5)$

$= (x - 5)(x^2 - 1)$

$= (x - 5)(x + 1)(x - 1)$

So, $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 = (x+1)(x-5)(x+1)(x-1)$.

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Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 2 & 8 & 0 & 2 & 8 \\ & & -8 & 0 & 0 & -8 \\ \hline & 2 & 0 & 0 & 2 & 0 \end{array}$$

2

$(x+4)(2x^3+2)$

$2(x+4)(x^3+1)$

$2(x+4)(x+1)(x^2-x+1)$

9. $p(x) = 3x^3 - 2x + 5; (x - 1)$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^3 + 2x^2 - 19x - 20$$

$\pm$ 1, 2, 4, 5, 10, 20

1)

1	2	-19	-20	
	1	3	-16	
1	3	-16	-36	

2)

1	2	-19	-20	
	2	8	-22	
1	4	-11	-42	

4)

1	2	-19	-20	
	4	24	20	
1	6	5	☺	

$(x-4)(x^2+6x+5)$

$= (x-4)(x+5)(x+1)$

$\boxed{x = 4, -5, -1}$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

$\pm \frac{1, 2, 3, 4, 6, 8, 12, 24}{1}$

1)

1	-4	-7	22	24
↓	1	-3	-10	12
1	-3	-10	12	$\boxed{36}$

2)

1	-4	-7	22	24
↓	2	-4	-22	0
1	-2	-11	0	$\boxed{24}$

3)

1	-4	-7	22	24
↓	3	-3	-30	-24
1	-1	-10	-8	$\boxed{0}$

$(x-3)(x^3 - x^2 - 10x - 8)$

4)

1	-1	-10	-8
↓	4	12	8
1	3	2	$\boxed{0}$

$(x-3)(x-4)(x^2+3x+2)$

$(x-3)(x-4)(x+2)(x+1)$

$\boxed{x = 3, 4, -2, -1}$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

$x(\cancel{x^2 - 2x - 8})\ (x-4)(x+2)$

$\pm \cancel{1, 2, 4, 8}$

$$\begin{array}{r|rrr} 2 & 1 & -2 & -8 \\ & & 2 & 0 \\ \hline & 1 & 0 & \boxed{-8} \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & -2 & -8 \\ & & 1 & -1 \\ \hline & 1 & -1 & \boxed{-9} \end{array}$$

$$\begin{array}{r|rrr} 4 & 1 & -2 & -8 \\ & & 4 & 8 \\ \hline & 1 & 2 & \boxed{0} \end{array}$$

$x(x-4)(x+2)$ $x = 4, -2, 0$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

$\pm \frac{1, 2, 4, 8}{1, 2} = \pm 1, 2, 4, 8, \frac{1}{2}$

4	2	-7	-8	14	8
	↓	8	4	-16	-8
	2	1	-4	-2	0

$(x-4)((2x^3+x^2)-(4x-2))$

$(x-4)(x^2(2x+1)-2(2x+1))$

$0 = (x-4)(2x+1)(x^2-2)$

$x = 4, -\frac{1}{2}, \pm\sqrt{2}$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

$\pm \frac{1, 2, 3, 6}{1}$

3	1	1	-14	6
	↓	3	12	-6
	1	4	-2	☹

$(x-3)(x^2+4x-2)$

$\boxed{x=3}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{16-4\cdot 1\cdot -2}}{2}$

$\frac{-4 \pm \sqrt{24}}{2}$

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3 2 2 2

$\frac{-4 \pm 2\sqrt{6}}{2}$

$\boxed{x = -2 \pm \sqrt{6}}$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1

$$p(x) = 2(x+2)(x-4)(x-1)$$