14-4 Solving Rational Equations

Book Section 9-3 pgs 498-504

Objectives:

I can solve a rational equation algebraically

I can identify extraneous solutions

I can solve real-world problems using rational equations

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(B) $\frac{2x-9}{x-7}+\frac{x}{2}=\frac{5}{x-7}$

Identify any excluded values.

$x-7=0$

$x \neq 7$

The excluded value is $x \neq 7$.

Identify the LCD.

The different factors are $2, (x-7)$

The LCD is $2(x-7)$

Multiply each term by the LCD. $\frac{2x-9}{x-7}\cdot 2(x-7)+\frac{x}{2}\cdot 2(x-7)=\frac{5}{x-7}\cdot 2(x-7)$

Divide out common factors. $\frac{2x-9}{\cancel{x-7}}\cdot \cancel{2}(x-7)+\frac{x}{\cancel{2}}\cdot \cancel{2}(x-7)=\frac{5}{\cancel{x-7}}\cdot \cancel{2}(x-7)$

Simplify. $2(2x-9)+x(x-7)=5(2)$

Use the Distributive Property. $4x-18+x^2-7x=10$

Write in standard form. $x^2-3x-28=0$

Factor. $(x-7)(x+4)=0$

Use the Zero Product Property. $x-7=0$ or $x+4=0$

Solve for x. $x=7$ or $x=-4$

The solution $x=7$ is extraneous because it is an excluded value. The only solution is $x=-4$.

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Example 1 Solve each rational equation algebraically.

(A) $\frac{3x+7}{x-5} = \frac{5x+17}{2x-10}$

$2(x-5)$

$x \neq 5$

LCD: $2(x-5)$

$\frac{3x+7}{x-5} \cdot 2(x-5) = \frac{5x+17}{2(x-5)} \cdot 2(x-5)$

$2(3x+7) = 5x+17$

$6x+14 = 5x+17$

$-5x \quad -14 \quad -5x \quad -14$

$x = 3$ ✓

Your Turn

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Solve each rational equation algebraically.

2. $\frac{8}{x+3} = \frac{x+1}{x+6}$

$x \neq -3, -6$

LCD: $(x+3)(x+6)$

$$\frac{8}{x+3} \cdot (x+3)(x+6) = \frac{x+1}{x+6} \cdot (x+3)(x+6)$$

$$8(x+6) = (x+1)(x+3)$$

$$8x + 48 = x^2 + 4x + 3$$

$$-8x \quad -48 \qquad -8x \quad -48$$

$$0 = x^2 - 4x - 45$$

$$0 = (x-9)(x+5)$$

$x = 9, -5$ ✓

Solve:

$$\frac{2}{x}-\frac{1}{6}=\frac{5}{2x}-\frac{1}{3}$$

$x \neq 0$

LCD: $6x$

$$\frac{2}{x}\cdot 6x-\frac{1}{6}\cdot 6x=\frac{5}{2x}\cdot 6x-\frac{1}{3}\cdot 6x$$

$$12-x=15-2x$$

$-12 \quad +2x \quad -12 \quad +2x$

$x=3$ ✓

Solve the rational equation algebraically pg. 504

$$\frac{x^2-29}{x^2-10x+21}=\frac{6}{x-7}+\frac{5}{x-3}$$

$x^2-10x+21$

$(x-7)(x-3)$

$x \neq 7, 3$

LCD: $(x-7)(x-3)$

$$\frac{x^2-29}{(x-7)(x-3)}(x-7)(x-3)=\frac{6}{x-7}(x-7)(x-3)+\frac{5}{x-3}(x-7)(x-3)$$

$$x^2-29=6(x-3)+5(x-7)$$

$$x^2-29=6x-18+5x-35$$

$$x^2-29=11x-53$$

$$-11x+53 \quad -11x+53$$

$$x^2-11x+24=0$$

$$(x-8)(x-3)=0$$

$x=8$ ✓, 3 ✗ extran.

10. Jake can mulch a garden in 30 minutes. Together, Jake and Ross can mulch the same garden in 16 minutes. How much time t, in minutes, will it take Ross to mulch the garden when working alone?

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$$\frac{240t}{1}\cdot\frac{1}{30}+\frac{1}{t}=\frac{1}{16}\cdot\frac{240t}{1} \qquad t\neq 0$$

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$2\cdot 3\cdot 5 \qquad t \qquad 2\cdot 2\cdot 2\cdot 2$

LCD: $2\cdot 3\cdot 5\cdot 2\cdot 2\cdot 2t$

$240t$

$8t+240=15t$

$-8t \qquad -8t$

$\frac{240}{7}=\frac{7t}{7}$

$t=34.3$ min.

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Your Turn

4. Kevin can clean a large aquarium tank in about 7 hours. When Kevin and Lara work together, they can clean the tank in 4 hours. Write and solve a rational equation to determine how long, to the nearest tenth of an hour, it would take Lara to clean the tank if she works by herself. Explain whether the answer is reasonable.

$x \neq 0$

LCD: $28x$

$\frac{28x}{7} + \frac{28x}{x} = \frac{28x}{4}$

$4x + 28 = 7x$

$-4x \quad -4x$

$\frac{28}{3} = \frac{3x}{3}$

$x = 9.3$ hours